Twin-width V : linear minors, modular counting, and matrix multiplication

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Twin-width is a recently introduced width parameter for graphs that aims at generalizing and unifying many previous notions from parameterized complexity.

I will first introduce the notion of *twin-decomposition*, a data structure encoding in space $\mathcal{O}(dn)$ in a tree-like fashion graphs having twin-width at most d, together with a certificate of their twin-width boundedness. We will observe that we can generalize the main result from [1], which states that there exists an algorithm that, given as input some first-order formula φ modelling a graph problem together with some graph G and some certificate that G has twin-width at most d, decides in time $\mathcal{O}(f(|\varphi|, d) \cdot n)$ whether G is a model of φ for some computable function f, by allowing to also make use of modular counting in the quantifiers involved in φ. From this perspective, $\exists^{i[p]} x, \psi(x)$ means "there exists i modulo p models of x satisfying $\psi(x)$". Combining this observation together with the approximation algorithm from [2] and the data structure introduced in [3], we observe that for every prime power q one can multiply two $n \times n$ matrices A, B of twin-width at most d and coefficients in the finite field $\mathbb{F}_q$ in time $\mathcal{O}_{d,q}(n^2 \log(n))$ without requiring any certificate of twin-width boundedness. However the dependency in d makes such an algorithm impractical. Our most technical result is an efficient algorithm that, given two twin-decompositions of A, B of width at most d, returns in time $q^{2d+o(d)} \cdot n$ a twin-decomposition of width $q^{d+o(d)}$ of AB.

Références

[1] É. Bonnet, E. Kim, S. Thomassé, R. Watrigant, *Twin-width I : Tractable FO Model Checking*, Journal of the ACM **69** (2022), 1–46.

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[3] J. Gajarský, M. Pilipczuk, W. Przybyszewski, S. Torunczyk, *Twin-Width and Types*, ICALP 2022 **229**, 123 :1–123 :21