N. Bousquet, F. Havet, N. Nisse, L. Picasarri-Arrieta, A. Reinald : Digraph redicolouring

Bousquet Nicolas, LIRIS, CNRS, Université Claude Bernard Lyon 1, Lyon, `nicolas.bousquet@univ-lyon1.fr`

Havet Frédéric, CNRS, Université Côte d'Azur, I3S, Inria, Sophia-Antipolis, `frederic.havet@inria.fr`

Nisse Nicolas, CNRS, Université Côte d'Azur, I3S, Inria, Sophia-Antipolis, `nicolas.nisse@inria.fr`

Picasarri-Arrieta Lucas, CNRS, Université Côte d'Azur, I3S, Inria, Sophia-Antipolis, `lucas.picasarri-arrieta@inria.fr`

Reinald Amadeus, LIRMM, Université de Montpellier, CNRS, Montpellier, `amadeus.reinald@lirmm.fr`

In this work, we generalize several results on graph recolouring to digraphs. A *k-dicolouring* of a digraph D is a partition of its vertex-set into k parts such that each part induces an acyclic subdigraph on D. The *k-dicolouring graph* of D is the graph whose vertices are the k-dicolourings of D and in which two k-dicolourings are adjacent if they differ on exactly one vertex. When the k-dicolouring graph of D is connected, then D is said to be *k-mixing*.

The *min-degeneracy* (respectively *max-degeneracy*), denoted by $\delta^*_{\min}$ (respectively $\delta^*_{\max}$), of a digraph D is the smallest k such that every subdigraph H of D has a vertex v with $\min\{d^+_H(v), d^-_H(v)\} \leq k$ (respectively $\max\{d^+_H(v), d^-_H(v)\} \leq k$).

We first show that every digraph D is k-mixing for all $k \geq \delta^*_{\min}(D) + 2$, generalizing a result from Bonsma, Cereceda and Dyyer. We also prove that, if D is an oriented graph, then D is k-mixing for all $k \geq \delta^*_{\max}(D) + 1$. We pose as a conjecture that the k-dicolouring graph of D has diameter at most $O(|V(D)|^2)$ when $k \geq \delta^*_{\min}(D)+2$. This is the directed analogue of Cereceda's conjecture for digraphs. We prove two results supporting this conjecture. We first prove that the k-dicolouring graph of D, when $k \geq 2\delta^*_{\min} + 2$, has diameter bounded by $(\delta^*_{\min}(D)+1)|V(D)|$, extending a result from Bousquet and Perarnau. We also prove that our conjecture is true when $k \geq \frac{3}{2}(\delta^*_{\min}+1)$, which generalizes a result from Bousquet and Heinrich.

Restricted to the special case of oriented graphs, we conjecture that every non 2-mixing oriented graph has maximum average degree at least 4. We prove, as a partial result, that every oriented graph with maximum average degree strictly less than $\frac{7}{2}$ is 2-mixing.