

# **J. Bensmail, H. Hocquard et P.M. Marcille : The Weak $(2, 2)$ -Labelling Problem for graphs with forbidden induced structures**

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The Weak  $(2, 2)$ -Conjecture is a graph labelling problem asking whether all connected graphs of at least three vertices can have their edges assigned red labels 1 and 2 and blue labels 1 and 2 so that any two adjacent vertices are distinguished either by their sums of incident red labels, or by their sums of incident blue labels. This problem emerged in a recent work aiming at proposing a general framework encapsulating several distinguishing labelling problems and notions, such as the well-known 1-2-3 Conjecture, a few of its variants, and so-called locally irregular decompositions. One further point of interest behind the Weak  $(2, 2)$ -Conjecture is that it is weaker than the 1-2-3 Conjecture, in the sense that the latter conjecture, if proved true, would imply the former one is true too.

We prove that the Weak  $(2, 2)$ -Conjecture holds for two classes of graphs defined in terms of forbidden induced structures, namely claw-free graphs and graphs with no pair of independent edges. One main point of interest for focusing on such classes of graphs is that the 1-2-3 Conjecture is not known to hold for them. Also, these two classes of graphs have unbounded chromatic number, while the 1-2-3 Conjecture is mostly understood for classes with bounded and low chromatic number.