Directed Acyclic Hypergraph Disjoint Clustering Problem with Path-length awarness

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Let $\mathcal{H} \stackrel{\text{def}}{=} (\mathcal{V}, \mathcal{A}, \mathcal{W}_v, \mathcal{W}_a)$ be defined as a set of vertices $\mathcal{V}$ and a set of hyperarcs $\mathcal{A}$, with a vertex weight function $\mathcal{W}_v : \mathcal{V} \to \overrightarrow{\mathbb{Z}}$ and a hyperarc weight function $\mathcal{W}_a : \mathcal{V} \to \overrightarrow{\mathbb{Z}}$ where each hyperarc $a \in \mathcal{A}$ is a subset of vertex set $\mathcal{V}$: $a \subseteq \mathcal{V}, \forall a \in \mathcal{A}$. Let us define $\mathcal{V}^R$ the red vertex set, and $\mathcal{V}^B$ the black vertex set, such that $\mathcal{V}^R \cap \mathcal{V}^B = \emptyset$ and $\mathcal{V}^R \cup \mathcal{V}^B = \mathcal{V}$. $\mathcal{H}$ is a directed acyclic hypergraph (DAH) with source and sink vertices in $\mathcal{V}^R$. Since a hypergraph can contain multiple DAHs in the general case, it is possible to represent this set of DAHs by a red-black hypergraph. Let $\mathbf{H} \stackrel{\text{def}}{=} \{\mathcal{H}_i, i \in \{1, n\}\}$ be a set of DAHs in which every $\mathcal{H}_i$ is a DAH. An example can be found in Fig. .

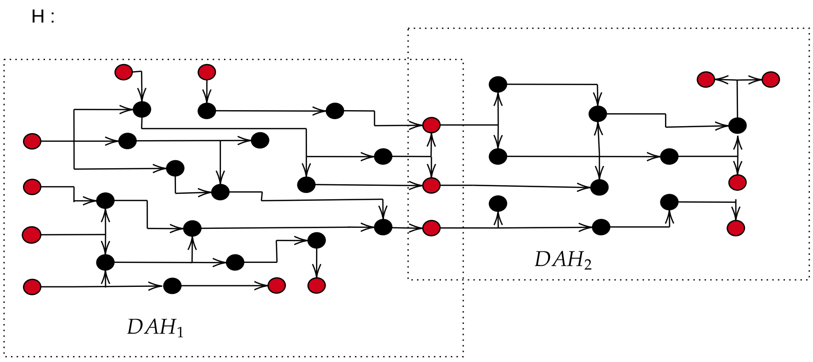

Figure 1: Hypergraph with 2 DAHs

A cluster is a subset of vertices such that each path-length passing through a cluster is penalized by a constant.

The problem $CN{<}\mathcal{V}, M, \Delta{>}$ consists in finding a clustering of the vertices of a DAG such that the clusters sizes are bounded by M and the longest path is minimized. We present an extension of $CN{<}\mathcal{V}, M, \Delta{>}$ defined for DAGs in [1] to directed acyclic hypergraphs and more generally to red-black hypergraphs. We will also present a binary search approach to approximate a solution taking into account the orientation properties of the paths.

[1] Z. Donovan, K. Subramani, and V. Mkrtchyan. *Disjoint Clustering in Combinatorial Circuits*, pages 201–213. 07 2019.