M. Caoduro and A. Talon: Finding small rectangle graphs with interesting properties

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The *boxicity* of a graph is the minimum dimension d such that the graph is the intersection graph of a family of axis-parallel boxes in $\mathbb{R}^d$. The problem of deciding the boxicity of a graph is $\mathcal{NP}$-hard, even for $d = 2$. Even though efficient algorithms are not likely to exist, the research on the subject necessitates developing strong enough methods for executing this task for small concrete graphs or, hopefully, relevant classes of graphs.

An equivalent definition of boxicity states that a graph G has boxicity d if its complement $\overline{G}$ is the union of d co-interval graphs. Since co-interval graphs can be recognized in linear time, this definition gives us an exponential time algorithm to identify graphs with boxicity at most d.

In this work, we focus on *rectangle graphs*, which are graphs with boxicity at most 2. Thanks to both algorithmic and more technical optimizations, we significantly sped up this algorithm, allowing us to recognize rectangle graphs of small order in a reasonable time.

Studying concrete graphs, even of small order, having invariants (such as the stability number, the clique covering number, the chromatic number, or the clique number) with "interesting" values can help us to refute or find tight examples for conjectures on rectangle graphs, like Wegner's conjecture or its analogous for the chromatic number.

Since triangle-free graphs are often used to find tight examples of similar geometric problems, we start our search here. So far, our main result is the following: *The smallest triangle-free* 4*-chromatic rectangle graph has* 15 *vertices.*

This implies that every triangle-free rectangle graph with at most 14 vertices is 3-colorable. In particular, some popular graphs with a small order, such as the Petersen graph, the Grötzsch graph (the smallest triangle-free, 4-chromatic graph), and Chvátal graph (the smallest triangle-free, 4-chromatic, 4-regular graph) are not rectangle graphs. This property for the Petersen graph and Grötzsch graph already appeared in the literature as a claim without proof.

This talk will also be the occasion to present a selection of open problems on rectangle graphs.