($\overrightarrow{P_6}$, *triangle*)**-free digraphs have bounded dichromatic number**

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The dichromatic number of a digraph is the minimum size of a partition of its vertices into acyclic induced subdigraphs. This generalization of the undirected case, coined by Neumann-Lara in 1982 [1], proved to be fruitful, as shown by the number of theorems and conjecture that translated well from the undirected case to the directed case. One such conjecture is the infamous (and largely open) Gyárfás-Sumner conjecture : in 2021, Aboulker, Charbit and Naserasr found an analogue conjecture in the directed case [2].

One of the open case states that, for any orientation of a path P and any integer k, digraphs with clique number at most k and not containing P as an induced subdigraph should have bounded dichromatic number. This remains largely open as, even when $k = 3$, it was only solved whenever P is any orientation of a path on at most 4 vertices [3] (in which case, it is actually solved for any value of k).

We prove that digraphs with no induced directed path on six vertices and no triangle have bounded dichromatic number.

Références

[1] V. Neumann-Lara, *The dichromatic number of a digraph*, *Journal Of Combinatorial Theory, Series B.* ***33****, 265-270 (1982)*

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[3] L. Cook, T. Masařík, M. Pilipczuk, A. Reinald and U. Souza, Proving a directed analogue of the Gyárfás-Sumner conjecture for orientations of P_4. *(arXiv,2022)*