Local certification and graph coloring

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Consider a graph property, say k-colorability. We want to check whether the graph has this property or not, but only with access to local information and with the help of some label (or certificate) on the vertices. Let us describe this more precisely.

Let G be a graph. A *labeling of size s* on $V(G)$, the vertex-set of G, is a function $\ell : V(G) \to \{0,1\}^s$. A *local algorithm* is an algorithm as follows: on every vertex of G, it takes as input the the label of the vertex and the multiset of labels of its neighbors, and outputs accept or reject.

We say that a property P is *locally certifiable* by labels of size s if there exists a local algorithm $\mathcal{A}$ such that:

- if a graph G has property P, then there exists a labeling of size s of $V(G)$ such that $\mathcal{A}$ accepts on every vertex of G,
- if a graph G does not have property P, then for every labeling of size s of $V(G)$, the algorithm $\mathcal{A}$ rejects on at least one vertex.

In this talk we are interested in k-colorability. We first see why labels of size $\lceil \log k \rceil$ are enough to certify k-colorabily, and then we address the open problem whether we can certify k-colorability with labels of smaller size, and show that we cannot certify k-colorabily, for $k \geq 3$, with labels of length 1.

If there is time, we also talk about the same theorem in the case that each vertex has also an identifier and it has access to its neighbors identifier as well as its own. This model is known as the *proof-labeling scheme*.

This talk is based on [1].

References

[1] Ardévol Martínez, V., Caoduro, M., Feuilloley, L., Narboni, J., Pournajafi, P., Raymond, J.F., *Lower bound for constant-size local certification*, arXiv:2208.14229