C. Dallard, V. Lozin, M. Milanič, K. Štorgel, V. Zamaraev : Graph functionality and symmetric difference

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Let G be a graph and A its adjacency matrix. A vertex $y \in V(G)$ is a function of $x_1, \ldots, x_k \in V(G)$ if $A(y, z)$, for any $z \in V(G) \setminus \{y, x_1, \ldots, x_k\}$, can be determined from $A(x_1, z), \ldots, A(x_k, z)$. Informally, the adjacency of y with any $z \in V(G)$ can be determined from the adjacency of $x_1, \ldots, x_k$ with z. The *functionality of* y, denoted $fun(y)$, is the minimum k such that y is a function of k other vertices. By extension, the *functionality of* G, denoted $fun(G)$, is the minimum k such that every induced subgraph of G contains a vertex of functionality at most k. It is known that bounded functionality strictly generalizes bounded treewidth and bounded clique-width, and that bounded VC-dimension is more general than bounded functionality.

The *symmetric difference of two vertices* $x, y \in V(G)$, denoted by $sd(x, y)$, is the size of the symmetric difference of $N(x)\setminus\{y\}$ and $N(y)\setminus\{x\}$. We can lift this definition to the whole graph by defining the *symmetric difference of* G as the minimum k such that every induced subgraph of G containing at least 2 vertices also contains a pair of distinct vertices whose symmetric difference is at most k. It is easily observed that bounded symmetric difference implies bounded functionality. Note that bounded twin-width also implies bounded symmetric difference.

In this talk, we will define the notions of functionality and symmetric difference of graphs, and present some known and new results regarding the boundedness of these parameters for some classes of intersection graphs, in particular in the case where one is bounded and the other unbounded.