Mónika Csikós and Nabil H. Mustafa : An improved algorithm to create low-crossing matchings

Mónika Csikós, IRIF, Université Paris Cité, csikos@irif.fr
Nabil H. Mustafa, LIPN, Université Sorbonne Paris Nord,
nabil.mustafa@univ-paris13.fr

Given a hypergraph $H = (V, \mathcal{E})$, the problem is to find a perfect matching of V such that each hyper-edge in $\mathcal{E}$ crosses few edges of the matching. This structure was originally introduced for geometric range searching in the 1980s and since then, it has found applications in various fields, including algorithmic graph theory and combinatorial data approximation.

The seminal work of Chazelle and Welzl [1] provided an algorithm that for any hypergraph with n vertices, m edges, and dual VC-dimension d, constructs a matching with crossing number $O(n^{1-1/d})$ in time $O(n^3 m)$. Since then, despite several advances for geometric hypergraphs, the general algorithm for abstract hypergraphs remained unimproved.

We propose a new sampling-based algorithm which is applicable to any finite hypergraph with dual VC-dimension d and provides a matching with expected crossing number $O(n^{1-1/d})$ in time $O(n^{1/d}m + n^{2+1/d})$ resulting in an $n^{2-1/d}$ factor speed-up on the construction time (assuming $m \geq n$).

Références

[1] Chazelle, B. and Welzl, E. *Quasi-Optimal Range Searching in Spaces of Finite VC-Dimension*,Discrete Comput. Geom. **23** (1989), 467—489.